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Abstract

This paper introduces a new scheme for the identification of multivariate behavioral macromodels from tabulated frequency-domain data. The method produces closed-form parametric expressions that reproduce with excellent accuracy the external port behavior of the structure, both as function of frequency and one or more external parmeters. The numerical robustness of the main algorithm is demonstrated on two significant examples.

Introduction and motivations

Model-based design is a standard practice in many different application areas. From initial concept to product finalization, various modeling steps take place, in order to choose the best candidate for prototype construction, testing, and final production. This is particularly true in the design of electrical interconnect structures, where system complexity calls for a fully automated design and optimization workflow.

Even for simple interconnect structures, many free variables must be determined in order to optimize the system performance. Examples can be geometrical parameters such as interconnect width and spacing or substrace height, or material parameters such as conductivity or permittivity. The particular combination of such parameters leading to best system performance is usually determined through lenghty optimization runs, often resorting to full-wave electromagnetic tools for the computation of the electrical response for any given parameter configuration.

This paper proposes a technique for the identification of multivariate macromodels. These can reproduce with excellent accuracy the response of a structure as a function of both frequency and external parameters within a prescribed range. Only a limited number of tabulated frequency responses at discrete parameters values is needed for the indentification of the parametric macromodel. A simple closed-form interpolation rule is then applied to recover the model response for any arbitrary parameters configuration. Since the model representation is rational in the frequency-domain, a parameterized SPICE-compatible model synthesis is possible, thus enabling fast optimization runs and what-if analyses in a CAD environment. This paper buils on preliminary results documented in [1]. Two major improvements are introduced, leading to robust and efficient numerical schemes.

Parametric formulation of electrical macromodels

We consider a linear system with a frequency response $H(j\omega, \lambda)$ which is also function of a design parameter λ. For the sake of simplicity, we focus on a scalar response with a single parameter, although the technique is general and applicable to the multivariate multiport case [1]. A set of samples of the frequency response is given[1]

$$H_{kl} = H(j\omega_k, \lambda_l) \qquad k = 1, \ldots, \bar{k} \quad l = 1, \ldots, \bar{l} \quad (1)$$

for different frequencies $\omega = \omega_k$ and parameter values $\lambda = \lambda_l$. These data can be the outcome of independent measurements or full-wave simulations performed for fixed values $\lambda = \lambda_l$ of the parameter within the range $[\underline{\lambda}, \bar{\lambda}]$.

The first step in the identification of a parametric model $\hat{H}(j\omega, \lambda)$ from (1) is the definition of its functional form. It turns out that the choice of a direct parameterization of poles and residues is not appropriate, due to the highly non-smooth behavior of the poles with respect to the parameter λ. A suitable formulation for parametric models was proposed in [1], based on the following general property of lumped electrical networks [2].

Theorem 1. *Let λ be the value of a resistor, inductor or capacitor in an arbitrary lumped circuit. Then, any network transfer function, including Z, Y, and S representations, can be written as a ratio of polynomials*

$$H(s, \lambda) = \frac{\sum_n (Q_{n0} + Q_{n1}\lambda) s^n}{\sum_n (q_{n0} + q_{n1}\lambda) s^n}, \quad (2)$$

where numerator and denominator coefficients are linear in λ.

This closed-form is valid independently on the system order and complexity. Moreover, the coefficients are linear in λ, i.e., they have the smoothest and simplest dependence on the parameter. In principle, this representation does not hold for generic parameters such as geometrical and material properties of interconnects or passive components. However, its generalization to higher order polynomials is expected to provide an excellent uniform approximation whenever the frequency response has a smooth dependence on the parameters. The adopted model representation is therefore

$$\hat{H}(s, \lambda) = \frac{\sum_n (Q_{n0} + Q_{n1}\lambda + \ldots Q_{n\bar{m}}\lambda^{\bar{m}}) s^n}{\sum_n (q_{n0} + q_{n1}\lambda + \ldots q_{n\bar{m}}\lambda^{\bar{m}}) s^n} = \frac{\sum_{n,m} Q_{nm}\psi_{nm}(s, \lambda)}{\sum_{n,m} q_{nm}\psi_{nm}(s, \lambda)} \quad (3)$$

where $\psi_{nm}(s, \lambda) = \lambda^m s^n$ denote the basis functions used in the expansion of numerator and denominator.

Model identification from tabulated data

This section proposes a numerical procedure for the estimation of coefficients Q_{nm} and q_{nm} so that the parametric

[1] The minimum and maximum values of a quantity n are denoted, respectively, as $\underline{n}$ and $\bar{n}$; sums are written as $\sum_n$, with the index varying from 0 up to $\bar{n}$.

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model (3) accurately fits the given samples (1). The objective is to minimize the data to model error defined as

$$\mathcal{E}^2 = \sum_{k=1}^{\bar{k}} \sum_{l=1}^{\bar{l}} \left| \hat{H}(j\omega_k, \lambda_l) - H_{kl} \right|^2 . \quad (4)$$

The numerical minimization of $\mathcal{E}$ is a very challenging task, for two main reasons. First, the presence of the unknowns q_{nm} at denominator of (3) makes the minimization problem nonlinear. Therefore, a direct numerical solution is inefficient and possibly affected by the local minima issue. Second, the possibly high-order powers of s and λ in (3) may lead to strong ill-conditioning, thus preventing the convergence of the numerical computations. Our proposed solution to these problems is now illustrated.

For the non-parametric case both issues are effectively addressed by the well-known Vector Fitting (VF) algorithm [3]. A direct extension of VF to the problem under consideration does not appear to be feasible, since it would imply an explicit parameterization of the poles, which we want to avoid. A possible alternative is based on a generalization to the parametric case of the Sanathanan-Koerner (SK) iteration [4], as suggested in [1]. This paper introduces several improvements, allowing for better accuracy and robustness.

We first consider the ill-conditioning problems arising from the polynomial nature of (3). This problem can be effectively solved by choosing a different set of basis functions. Therefore, we rewrite (3) as

$$\hat{H}(s, \lambda) = \frac{N(s, \lambda)}{D(s, \lambda)} = \frac{\sum_{n,m} R_{nm} \phi_{nm}(s, \lambda)}{\sum_{n,m} r_{nm} \phi_{nm}(s, \lambda)} , \quad (5)$$

where the new basis functions are defined as first order rational functions of frequency

$$\phi_{nm}(s, \lambda) = \begin{cases} T'_m(\lambda) & \text{for } n = 0 \\ T'_m(\lambda)(s - a_n)^{-1} & \text{for } n \neq 0 \end{cases} \quad (6)$$

where

$$T'_m(\lambda) = T_m \left(2 \frac{\lambda - \underline{\lambda}}{\overline{\lambda} - \underline{\lambda}} - 1 \right) \quad (7)$$

is the m-th order Chebychev polynomial applied to the rescaled parameter λ, and where the poles a_n are fixed. Such poles a_n can be linearly distributed over the bandwidth $[\omega_1, \omega_{\bar{k}}]$ spanned by (1), as in routine VF applications. A better solution is obtained by applying standard VF to the system frequency response for a fixed value of λ. We adopt this second choice, since it enhances the convergence speed of the overall fitting process.

Substituting (5) into (4), we get

$$\mathcal{E}^2 = \sum_{k=1}^{\bar{k}} \sum_{l=1}^{\bar{l}} \left| \frac{\sum_{n,m} \left[R_{nm} - r_{nm} H_{kl} \right] \phi_{nm}(j\omega_k, \lambda_l)}{\sum_{n,m} r_{nm} \phi_{nm}(j\omega_k, \lambda_l)} \right|^2 , \quad (8)$$

which is still nonlinear in the unknowns r_{nm}. The SK iteration [4] provides a way to linearize the error minimization problem. In essence, (8) is solved iteratively. At each iteration i, the unknown denominator is replaced by its known estimate provided by previous iteration $i - 1$

$$\mathcal{E}_i^2 = \sum_{k=1}^{\bar{k}} \sum_{l=1}^{\bar{l}} \left| \frac{\sum_{n,m} \left[R_{nm}^{(i)} - r_{nm}^{(i)} H_{kl} \right] \phi_{nm}(j\omega_k, \lambda_l)}{\sum_{n,m} r_{nm}^{(i-1)} \phi_{nm}(j\omega_k, \lambda_l)} \right|^2 . \quad (9)$$

The iterative scheme is setup with $r_{00}^{(0)} = 1$ and $r_{nm}^{(0)} = 0 \quad \forall n, m > 0$. It can be shown that the resulting linear least squares problem is unbiased and asymptotically equivalent to (8). Standard and robust numerical techniques are available for the minimization of (9). The benefits of proposed approach are outlined in Table 1, which reports the condition number of the first SK iteration for different choices of basis functions. Good numerical performance is insured when the condition number is as small as possible, condition fulfilled by the proposed technique.

Frequency	Parameter	Condition number
Polynomial	Polynomial	6.9×10^{13}
Rational	Polynomial	5.3×10^{5}
Rational	Chebychev	1.3×10^{5}

Table 1: Condition number of first SK iteration for polynomial (3), rational-polynomial [1] and proposed rational-Chebychev (6) basis functions.

Normalization of the model coefficients

A close inspection of (9) reveals that the SK iteration admits the trivial solution $R_{nm}^{(i)} = r_{nm}^{(i)} = 0, \forall n, m$. Therefore, some non-triviality constraint must be added. It turns out that the specific choice of this constraint has a dramatic influence on the quality of the results. The simplistic solution of fixing one of the model coefficients, e.g. $r_{00} = 1$ as used in [1], may slow down or even prevent the convergence of the SK iteration. In this paper, we propose a better normalization scheme that guarantees improved performance both in terms of accuracy and convergence speed. This normalization is inspired to the relaxed normalization developed for VF [5]. It is established by adding to the least squares problem for the minimization of (9) the constraint

$$\sum_{k,l} \mathbb{R}\mathrm{e} \left\{ \frac{\sum_{m,n} (r_{nm}^{(i)} - 1) \phi_{nm}(j\omega_k, \lambda_l)}{\bar{k}\bar{l} \sum_{m,n} r_{nm}^{(i-1)} \phi_{nm}(j\omega_k, \lambda_l)} \right\} = 0 . \quad (10)$$

This constraint is enforced in least squares sense and guarantees a non-vanishing weighted average of the coefficients $r_{nm}^{(i)}$, thus making (5) well-defined at all iterations.

Uniform stability of parametric models

Stability is a fundamental property for macromodels, requiring all model poles p_n to be in the left half plane. For parametric models like (5), uniform stability within the entire parameter range implies

$$\mathbb{R}\mathrm{e} \{ p_n(\lambda) \} < 0 \quad \forall \lambda \in [\underline{\lambda}, \overline{\lambda}] . \quad (11)$$

Since poles are only indirectly evaluated from (5), checking and enforcing (11) is a difficult task.

A direct test of (11) can be performed only with a brute force approach, by gridding the interval $[\underline{\lambda}, \bar{\lambda}]$ at many points and checking (11) at each location. Although being simple, this solution is not optimal, since it may be unable to detect stability violations localized in a very small subinterval of $[\underline{\lambda}, \bar{\lambda}]$. Or, it may be time consuming, if the number of grid points or parameters is large. Therefore, better mathematical conditions for uniform stability are highly desirable, to test stability and more importantly to develop new fitting algorithms with an automatic stability enforcement. These conditions are presented below.

We start by writing the denominator $D(s, \lambda)$ of (5) as

$$D(s, \lambda) = r_0(\lambda) + \boldsymbol{c}(\lambda) \left(s\mathbf{I} - \boldsymbol{\Delta}\right)^{-1} \boldsymbol{b}\,, \qquad (12)$$

where

$$r_n(\lambda) = \textstyle\sum_m r_{nm} T'_m(\lambda), \qquad \boldsymbol{b} = [1, \ldots, 1]^T, \qquad (13)$$

$$\boldsymbol{c}(\lambda) = [r_1(\lambda), \ldots, r_{\bar{n}}(\lambda)], \qquad \boldsymbol{\Delta} = \mathrm{diag}(a_n)\,, \qquad (14)$$

The poles $p_n(\lambda)$ of (5) are the roots of $D(s, \lambda)$, since the a_n defining the basis functions cancel out. A direct calculation shows that $p_n(\lambda)$ can be obtained as the eigenvalues of

$$\mathbf{A}(\lambda) = [r_0(\lambda)\boldsymbol{\Delta} - \boldsymbol{b}\boldsymbol{c}(\lambda)]/r_0(\lambda)\,. \qquad (15)$$

Therefore, uniform stability is guaranteed if and only if $\mathbf{A}(\lambda)$ has all eigenvalues in the left half plane, i.e. is a stable matrix.

Some results are available for checking the stability of parameter dependent matrices. For the case of degree $\bar{m} \leq 1$, we have that the stability of $\mathbf{A}(\lambda)$ can be conveniently verified through linear matrix inequalities (LMI), as stated by the following theorem [6].

Theorem 2. *If $r_0(\lambda)$ is not vanishing[2] in $[\underline{\lambda}, \bar{\lambda}]$, $\mathbf{A}(\lambda)$ is stable for all $\lambda \in [\underline{\lambda}, \bar{\lambda}]$ if a symmetric and positive definite matrix $\mathbf{P}$ exists such that*

$$\mathbf{A}^T(\underline{\lambda})\mathbf{P} + \mathbf{P}\mathbf{A}(\underline{\lambda}) < 0\,, \quad \mathbf{A}^T(\bar{\lambda})\mathbf{P} + \mathbf{P}\mathbf{A}(\bar{\lambda}) < 0\,. \qquad (16)$$

Therefore, (5) is stable if a positive definite solution for (16) exists, as can be numerically verified using convex optimization techniques. Moreover, Theorem 2 states important conditions that will be exploited in our future works to enforce model stability by construction.

The case of $\bar{m} > 1$ is more complicated, since results like Th. 2 involving only the extremes of the parameter range cannot exist [7]. Stability conditions for this case, like those reported in [6], will be subject of future investigations.

Application examples

The first application example concerns a via with stub, depicted in Fig. 1, connecting a microstrip line and a stripline in a multilayer PCB. When the via is created, the metallization runs from top to bottom. The bottom part is unnecessary to electrically connect the two lines and can be a potential source of Signal Integrity problems. To mitigate these problems, the stub length h can be adjusted through backdrilling. Our objective is to generate a macromodel for the 2-port via structure preserving the dependence on the free parameter h.

[2] In the linear case $\bar{m} \leq 1$, $r_0(\lambda) \neq 0$ for $\lambda \in [\underline{\lambda}, \bar{\lambda}]$ if and only if $r_0(\underline{\lambda})$ and $r_0(\bar{\lambda})$ have the same sign [7].

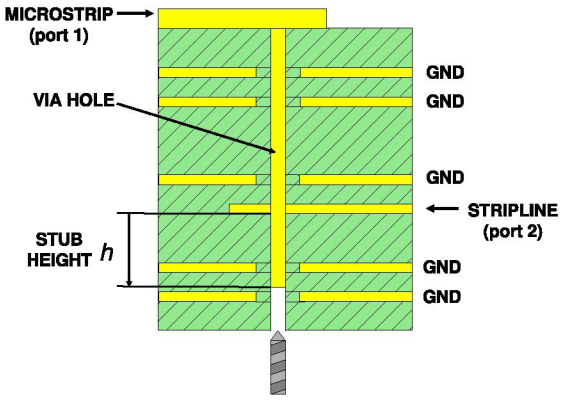

Figure 1: Backdrilled via cross section.

The scattering parameters of the via were obtained using an EM solver. Several stub heights $h = 0$, 200, 400, 478, 606, 716 μm were considered, including both full length ($h = 716\,\mu$m) and completely backdrilled ($h = 0\,\mu$m) stub. The parametric SK iteration was applied to compute the macromodel. First, the model was normalized by setting $r_{00} = 1$ in (5). Unfortunately, this compromised the model identification, either because of convergence failure or low accuracy. Instead, using the proposed algorithm with normalization (10), we obtained an accurate macromodel with order $\bar{n} = 14$ and degree $\bar{m} = 3$. The model was computed in 2.5 minutes on a laptop with a 1.83 GHz processor.

To validate the model quality in the whole parameter range, additional frequency responses were computed for the new parameter values $h = 100$, 300, 418, 496 μm. The model responses at these values were computed by evaluating the closed-form expression (5). Figure 2 shows a model vs data comparison for all (reference and validation) values of h, showing a very good agreement with a maximum deviation of 2.2×10^{-2}. Although stability is not guaranteed by the SK algorithm, the computed model is stable.

As a second test case, we consider a 2-turns integrated spiral inductor (courtesy of MSDT Consortium, Georgia Tech). The trace width w and the substrate dielectric constant ε_r were considered as parameters. The S_{11} of the inductor was computed with a full-wave solver from 0.1 up to 15 GHz for $w = 3, 4, 5, 6, 7$ mils and $\varepsilon_r = 30, 35, 40$. All available data were used to fit the parametric model, except for the responses corresponding to $w = 4, 6$ mils and $\varepsilon_r = 35$, reserved for validation purposes. In Fig. 3, the response of a model (order 6, degree 2 with respect to w and 1 with respect to ε_r) is compared to the raw data. A very good match can be observed, since the maximum modeling error is only 4×10^{-3} (0.4%). Also in this example the relaxed normalization (10) was used, leading to a significantly higher model accuracy. The required computation time for this example was only 2 s. Also in this case the model poles are stable, as shown in Fig. 4.

Conclusions

This paper introduced some new developments aimed at the automated construction of multivariate frequency- and

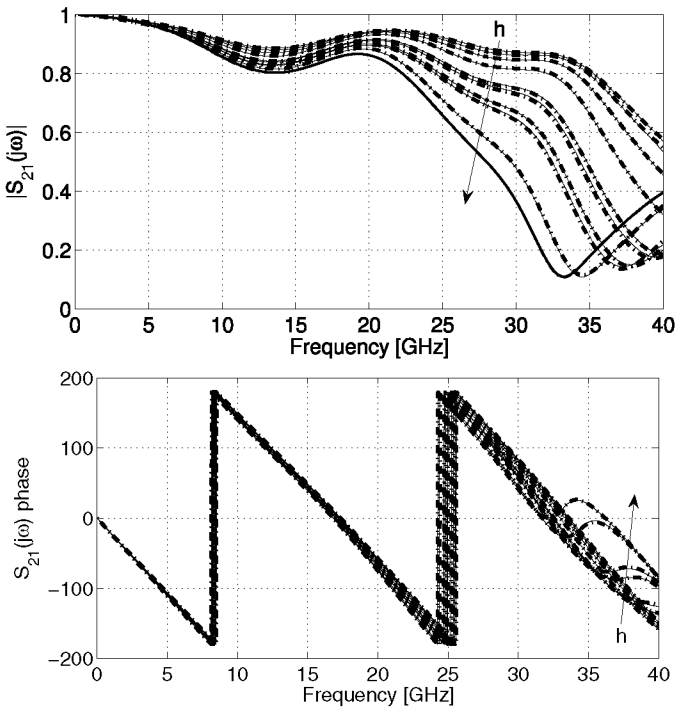

Figure 2: Magnitude and phase of S_{21} of the via macromodel. Raw frequency data (solid line) and model responses (dash dot line) are depicted for $h = 0$, 100, 200, 300, 400, 418, 478, 496, 606, 716 μm.

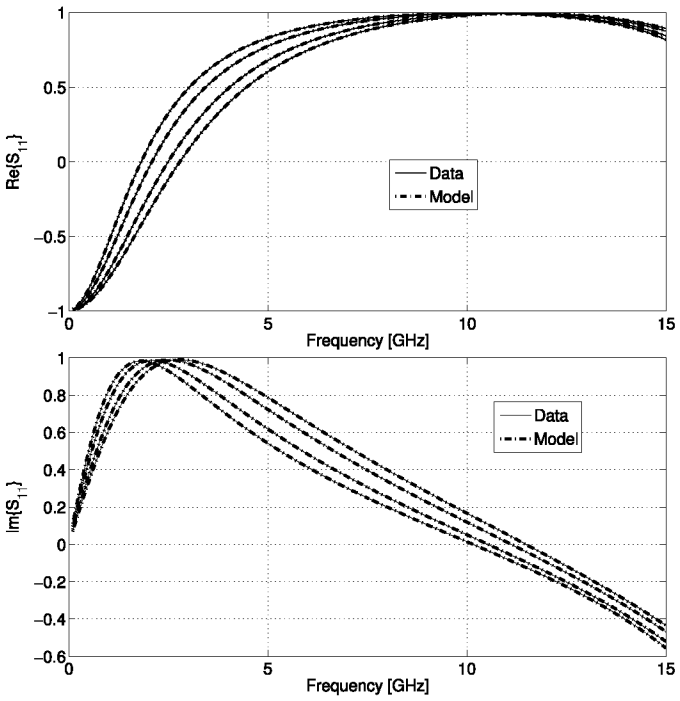

Figure 3: Macromodel of a spiral inductor. Model responses (dash-dot lines) and raw data (solid lines) are compared for different combinations of w (3, 4, 6, 7 mils) and ε_r (40, 35, 35, 30).

parameter-dependent macromodels. The theoretical formulation and the devised numerical identification scheme have been demonstrated to be sound and effective in modeling real-world interconnects and passive components.

Acknowledgments

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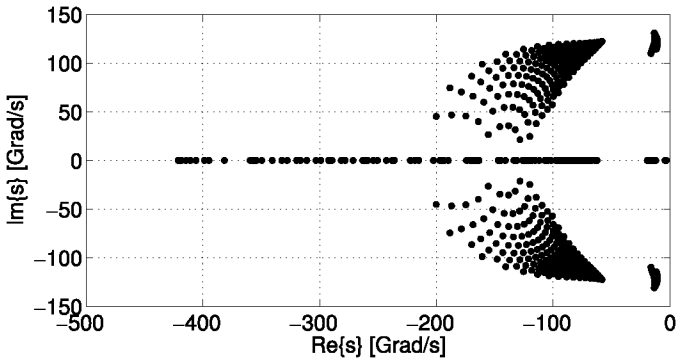

Figure 4: Poles of the inductor macromodel for $w \in [3, 7]$ mils and $\varepsilon_r \in [30, 40]$.

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